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Massive supergravity and deconstruction

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Abstract: We present a simple superfield lagrangian for massive supergravity. It comprises the minimal supergravity lagrangian with interactions as well as mass terms for the metric superfield and the chiral compensator. This is the natural generalization of the Fierz-Pauli lagrangian for massive gravity which comprises mass terms for the metric and its trace. We show that the on-shell bosonic and fermionic fields are degenerate and have the appropriate spins: 2, 3/2, 3/2 and 1. We then study this interacting lagrangian using goldstone superfields. We find that a chiral multiplet of goldstones gets a kinetic term through mixing, just as the scalar goldstone does in the non-supersymmetric case. This produces Planck scale ($M_{\rm Pl}$) interactions with matter and all the discontinuities and unitarity bounds associated with massive gravity. In particular, the scale of strong coupling is $(M_{\rm Pl}m^4)^{1/5}$, where m is the multiplet's mass. Next, we consider applications of massive supergravity to deconstruction. We estimate various quantum effects which generate non-local operators in theory space. As an example, we show that the single massive supergravity multiplet in a 2-site model can serve the function of an extra dimension in anomaly mediation.

Keywords: Field Theories in Higher Dimensions, Supersymmetry Breaking, Supergravity Models, Superspaces.

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1. Introduction

Only a handful of papers have been written on massive supergravity [1]–[5]. In fact, it seems that little is known beyond the free theory, and for good reason. *Massless* supergravity is complicated enough and at least it is constrained by powerful gauge symmetries which a mass must break. Moreover, the physical graviton is massless and the gravitino cannot be, so a theory in which the graviton and gravitino have degenerate nonzero mass cannot describe the real world. Thus, an interacting theory of massive supergravity may seem to be a difficult and phenomenologically irrelevant mathematical exercise. At least, that is the situation if one ignores recent developments in extra dimensions, supersymmetry breaking, massive gravity and deconstruction.

First, if there are compact extra dimensions and supersymmetry, then there will be massive supergravity multiplets at the compactification scale. Since it is conceivable that this scale will be accessible at future colliders, it would be nice to have a theory for the particles which may show up. As a first step in this direction, we write down an interacting supergravity theory containing massive and massless supergravity multiplets.

But even if the extra dimensions are very small, massive supergravity may play a crucial role in the low-energy theory. This is the case, for example, in models of anomaly mediation, in which supersymmetry breaking occurs on a hidden brane sequestered from

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the standard model by some distance in a fifth dimension [6]. The basic idea in these models is that dangerous couplings of the visible and hidden sectors are constrained by locality in the fifth dimension. It is natural to assume that such couplings are absent at tree level, and then loop corrections are appropriately suppressed by the size of the extra dimension. Of course, these quantum effects must be the same in the 4D effective theory obtained after integrating out the extra dimension. Therefore, in the 4D theory, 5D locality must manifest itself somehow in the regulation of divergent graviton exchange diagrams by massive KK states. To understand this, we need an interacting theory of massive supergravity. And then we can apply the methods of dimensional deconstruction [7]–[9] which have been developed for gauge theory. This is the subject of sections 7 and 8.

In addition to these phenomenologically inspired motivations, there are a number of more theoretical questions we may ask about massive supergravity. Massive (non-supersymmetric) gravity is a very peculiar theory, and we might hope that by adding supersymmetry we might gain insight into, or resolve some of these peculiarities. For example, there is a factor of 4/3 difference between tree level processes involving a massless graviton as compared to a graviton with an infinitesimally small mass [10, 11]. So we may ask: if light passed by a supersymmetric sun, would its angle of deflection change discontinuously as we take the mass of the supergravity multiplet to zero? In general, we would like to know whether each such classical phenomenon has a supersymmetric analog.

Massive gravity is also fascinating, theoretically, at the quantum level [12]. For example, if we try to discretize a gravitational dimension, consistency requirements in the quantum theory force us to keep the lattice spacing much larger than the Planck length [13, 14]. To approach the continuum limit, we must explicitly add interactions among distant lattice sites. One might hope that if supersymmetry is essential for regulating gravity, it would manifest its influence on some of these classical or quantum phenomena. In section 6 we consider this possibility, but show that adding supersymmetry does not make massive gravity any less eccentric.

Now, to answer any physical question about massive supergravity, we need an interacting lagrangian. As a start, we should look for a free theory guaranteeing the correct on-shell degrees of freedom [1]–[4]. We prefer to use superfields, as in [1], rather than than components because we expect the mass term to preserve global $N = 1$ supersymmetry. However, we will not use the lagrangians of [1]. These involve auxiliary fields not present in any known off-shell formulation of massless supergravity, and therefore they are difficult to generalize to the interacting case. In particular, it is hard to tell which symmetries are broken by the mass term because we do not know how all the fields transform under the symmetry group of supergravity. This issue will be explored in section 2, and we reproduce and examine the work of [1] in section 3.2.

We construct an improved linearized lagrangian in section 3 using the simple and powerful formalism of superspin projectors [15, 16]. Conveniently, it ends up being exactly the linearized lagrangian for old minimal supergravity with the addition of mass terms for the metric superfield H_m and the chiral compensator e^{Σ}. The compensator's mass requires the introduction of a real superfield P, which acts as a prepotential: $\Sigma = -\frac{1}{4}\bar{D}^2 P$. And the action, once we add the interactions of massless supergravity, can be written as:

$$S = \int d^8 z \left\{ \mathcal{E}^{-1}[H_m, \Sigma] + m^2 H_m^2 + \frac{9}{4} m^2 P^2 \right\}. \tag{1.1}$$

Here, $\mathcal{E}^{-1}[H_m, \Sigma]$ is the inverse of the supervierbein superdeterminant, written as a functional of H_m and Σ. For $m = 0$, (1.1) reduces to the action for massless supergravity [17, 18].

Already, one can see the relevance of massive supergravity to 5D physics. The action (1.1) shares many features with a recent parameterization of of 5D supergravity in terms of 4D superfields [19]. Indeed, this clearly written and practical paper inspired many aspects of our current approach. However, not only is the presentation in [19] confined to the free theory, but it includes additional auxiliary fields whose function in the 4D effective action is unknown. Refs. [3, 4], which derive a lagrangian from string theory, suffer from the same drawbacks. We found that in order to isolate the relevant degrees of freedom for *just* the massive supergravity multiplet, we had to rederive the free lagrangian from a 4D point of view. We will comment more on the additional fields in the section on deconstruction (section 7).

While the concise form of the action (1.1) has a certain utility, it completely obscures the propagating degrees of freedom. Nevertheless, after performing judicious field redefinitions and integrating out the auxiliary fields, the action reduces to:

$$\begin{aligned} S = \int d^4 x \sqrt{g} R[g] - \frac{1}{4} m^2 (h_{mn}^2 - h^2) - \\ - \frac{1}{4} F_{mn}^2 - \frac{1}{2} m^2 A_m^2 - \frac{1}{2} \varepsilon^{mnpq} \psi_m \gamma_n \partial_p \bar{\psi}_q + \frac{1}{4} m \bar{\psi}_m [\gamma_n, \gamma_m] \psi_n + \cdots . \end{aligned} \tag{1.2}$$

This contains the correct on-shell kinetic terms for a massive supergravity multiplet: a real spin-2, a real spin-1, and a complex Dirac spin-3/2 degree of freedom, all degenerate in mass. Indeed, the on-shell quadratic lagrangian is fixed completely by unitary and global supersymmetry, and so that part of (1.2) matches previous results [2]. The $\cdots$ are interactions. In contrast to the non-supersymmetric case, some of these interactions are proportional to m. We explain the significance of this is section 4, where the bosonic and fermionic component analysis of the linearized theory is worked out in full detail.

Note that the first line of (1.2) makes up the interacting Fierz-Pauli lagrangian for massive non-supersymmetric gravity. One can see that (1.1) is the natural supersymmetric generalization of Fierz-Pauli — the tuning of supersymmetric mass terms H_m^2 and P^2 to have the ratio 9/4 serves the same function as tuning the h_{mn}^2 and h^2 parts of Fierz-Pauli to have the ratio -1. These particular combinations eliminate the negative energy degrees of freedom. This can be shown through the equations of motion of either theory, but it is much clearer when we introduce goldstones to represent the dangerous modes. The goldstone formalism is especially useful because it reveals the scale at which these negative norm ghosts come in to haunt us. That is, it shows the energy scale at which the effective theory breaks down, and below which the tuning of the mass terms in Fierz-Pauli, and in (1) are technically natural. In both cases it is at energy $\Lambda_5 = (M_{\rm Pl} m^4)^{1/5}$.

The goldstones are the degrees of freedom that the massive theory has and the massless theory lacks. Essentially, in the massless theory, we can set them to zero by a gauge transformation. So to understand the goldstones, we need to comprehend and to parameterize the invariance of massless supergravity. There are many ways to do this, none of which we find to be particularly natural or transparent. In section 5 we discuss some of the relevant issues, and present the set of transformations which we have found to be most straightforward. This lets us isolate the strongest interactions, which turn out to be among a chiral multiplet of goldstones. This multiplet contains the scalar longitudinal mode of the graviton, as well as the longitudinal modes of A_m and $\bar{\psi}_m$ from (1.2).

In section 7 we show how to use the massive supergravity lagrangian (1.1) to deconstruct 5D supergravity theories. We work out the natural size of the relevant UV and IR dominated contributions to operators in the effective theory. This lets us see how well deconstruction can reproduce the effect of sequestering within the effective theory. Section 8 applies these rules to a deconstructed version of anomaly mediation.

Throughout this paper, we try to stick to the supersymmetry conventions of Wess and Bagger [20]. Also, to be clear, the terms *vector* and *scalar* refer to the external indices on a multiplet of field (V_m is a vector) while *spin-0* and *spin-1* refer to the irreducible representations of the Lorentz group. Contracted indices are often omitted if there is no ambiguity, for example $(\partial A) = \partial_m A_m$ and $(\partial\partial h) = h_{mn,m,n}$. We never use curved space notation; indices are raised and lowered with the flat space Minkowski metric η_{mn} and with $\sigma^m_{\alpha\dot{\alpha}}$.

2. Non-supersymmetric examples

Before we begin to justify equation (1.1), and study its properties, we will investigate some of the critical issues in a non-supersymmetric setting. To begin, recall the Fierz-Pauli lagrangian:

$$\mathcal{L}_{\mathrm{FP}} = \frac{1}{2}h_{mn}\Box h_{mn} + h_{mn,n}^2 + h(\partial\partial h) - \frac{1}{2}h\Box h - \frac{1}{2}m^2(h_{mn}^2 - h^2)\,. \tag{2.1}$$

This is the unique quadratic lagrangian constructed out of a single two-index tensor which propagates a spin-2 field and is free of tachyons and ghosts. It is conveniently related to the Einstein lagrangian expanded around flat space $g_{mn} = \eta_{mn} + h_{mn}$:

$$M_{\mathrm{Pl}}^2\sqrt{g}R - \frac{1}{4}M_{\mathrm{Pl}}^2 m^2(h_{mn}^2 - h^2) = \frac{1}{2}M_{\mathrm{Pl}}^2\mathcal{L}_{\mathrm{FP}} + \text{interactions}\,. \tag{2.2}$$

These interactions are in some sense arbitrary, as the lagrangian no longer has exact general-coordinate invariance (GC). In fact, (2.2) now leads to amplitudes which violate unitarity well below M_{Pl}, at energies $E \sim \Lambda_5 = (M_{\mathrm{Pl}}m^4)^{1/5}$. It could be better: one can raise the strong couping scale to $\Lambda_3 = (M_{\mathrm{Pl}}m^2)^{1/3}$ by adding additional terms to (2.2).[1] However, it could also be worse: if the tensor structure of the two derivative interactions were

[1] Λ_3 is the best one can do with a single spin-2 field. If the massive graviton is the first Kaluza-Klein mode of a 5D theory, the additional KK modes raise the strong coupling scale to $M_{5D} = \Lambda_{2/3}$.

completely arbitrary, that is, not protected by a custodial GC symmetry, the theory would lead to unitarity violation at $\Lambda_7 = (M_{\rm Pl} m^6)^{1/7} < \Lambda_5$. But if the interactions in (2.2) were arbitrary, we would have a bigger problem: when $m = 0$ the theory does not look like gravity! And so $M_{\rm Pl}$ has no physical interpretation.

This last point is worth emphasizing. It is essential that $\mathcal{L}_{\rm FP}$ looks like linearized Einstein gravity, with its linearized symmetries, for $m = 0$. Moreover, we also must be able to identify the auxiliary fields which appear in the massive sector so that we understand how the symmetries are broken. For example, consider the following lagrangian:

$$\begin{aligned} \mathcal{L}_A = & \frac{1}{2} h_{mn} \Box h_{mn} + h_{mn,n}^2 + h(\partial\partial h) - \frac{1}{2} h \Box h - \\ & - \frac{1}{2} m^2 h_{mn}^2 - m A_m (h_{mn,n} - h_{,m}) - \frac{1}{8}(A_{m,n} - A_{n,m})^2 + \frac{3}{4} m^2 A_m^2 \,. \end{aligned} \tag{2.3}$$

At first glance, this lagrangian seems to comprise a spin-2 and a spin-1 field. But observe that there is a bilinear coupling between them. In fact, there are only the on-shell degrees of freedom of a massive spin-2 field; the equations of motion force $A_m = h_{mn,n} = h = 0$. So this lagrangian is a perfectly viable alternative to $\mathcal{L}_{\rm FP}$ (2.1), at the quadratic level. However, when we embed the first line in $\sqrt{g}R$ and set $m = 0$, we are left with an additional free vector field with no obvious physical interpretation. Thus, when we turn the mass back on, we are stuck in unitary gauge. That is, we cannot study the interacting theory using goldstones, as we do not know what symmetry these goldstones are supposed to realize. Moreover, it is not possible to turn (2.3) into $L_{\rm FP}$ through some straightforward field redefinition.[2]

Now, in the supersymmetric case, it is somewhat easier to produce an analog of $\mathcal{L}_A$ (2.3) than of $\mathcal{L}_{\rm FP}$ (2.1). The difficulty arises because the metric is in the $\theta\sigma^n\bar{\theta}h_{mn}$ component of a real vector superfield H_m. So it is easy to write down the mass term $\int d^4\theta H_m^2 = \int d^4x h_{mn}^2 + \cdots$ but difficult to project out the trace. Thus, we can get all the quadratic h terms in L_A without much work, but not those of $\mathcal{L}_{\rm FP}$. The solution, of course, is to use the conformal compensator. In the non-supersymmetric case, it works as follows.

Start with $\mathcal{L}_{\rm FP}$ embedded in GR (2.2). We can rewrite it slightly by introducing an auxiliary scalar field s:

$$\mathcal{L}_s = 2\sqrt{g}R[g] - \frac{1}{2} m^2 h_{mn}^2 + 2m^2 s h - 2m^2 s^2 \,. \tag{2.4}$$

Setting s to its equation of motion reproduces (2.2). Then observe that

$$\begin{aligned} \mathcal{L}_s & = 2\sqrt{g}R[g] - \frac{1}{2} m^2 (e^{-2s} g_{mn} - \eta_{mn})^2 + 6m^2 s^2 + \cdots \\ & = 2\sqrt{e^{2s}\hat{g}}R[e^{2s}\hat{g}] - \frac{1}{2} m^2 \hat{h}_{mn}^2 + 6m^2 s^2 + \cdots \,, \end{aligned} \tag{2.5}$$

where $\hat{g}_{mn} = e^{-2s} g_{mn}$, $\hat{h}_{mn} = \hat{g}_{mn} - \eta_{mn}$, and $\cdots$ are cubic and higher order terms. The awkward trace term $m^2 h^2$ has been replaced by a mass term for the conformal compensator.

[2] It is possible to show that (2.1) and (2.3) are dual, in the sense that they are gauge fixed versions of a new mother lagrangian with an additional U(1) symmetry.

Moreover, the lagrangian is now Weyl invariant when we set $m = 0$. Of course, this Weyl invariance is a complete fake, as we can just use it to set $s = 0$. But clearly the form (2.5) suggests that we should be able to find a massive supergravity lagrangian with the features of (1.1): mass terms for the metric and conformal compensator, and conformal invariance in the massless limit.

3. Constructing the linearized theory

Without further introduction, we will now study the massive supergravity lagrangian. We will start by trying to derive a linearized lagrangian which is the supersymmetric version of $\mathcal{L}_{\rm FP}$. This is essentially the approach of [1]. But since we hope to embed our linear lagrangian in an interacting theory, we have the additional requirement that our lagrangian match some known formulation of supergravity when $m = 0$.

The most (mathematically) transparent way to construct lagrangians for higher-spin fields is through projectors [15, 16, 17, 18]. (See also [21] for a succinct application in the massless case). For example, the spin-0 and spin-1 projectors for a Lorentz vector are:

$$\omega_{mn} = \frac{\partial_m \partial_n}{\Box} \qquad \text{and} \qquad \theta_{mn} = \eta_{mn} - \omega_{mn} \, . \tag{3.1}$$

This assures us that a lagrangian like

$$\mathcal{L} = \frac{1}{2} A_m \Box \theta_{mn} A_n = -\frac{1}{4} F_{mn}^2 \tag{3.2}$$

contains only spin-1 degrees of freedom. Moreover, because the projectors are orthogonal ($\theta\omega = \omega\theta = 0$), this lagrangian automatically has a gauge invariance under $A \to A + \omega\delta A$. So the spin content and symmetries can basically be read off the projectors appearing in the lagrangian.

In supersymmetry, projectors are almost always used, whether or not they are acknowledged. For a general scalar superfield Ψ, we can isolate the linear ($\bar{D}^2\Psi_L = D^2\Psi_L = 0$), chiral ($\bar{D}\Psi_+ = 0$) and antichiral ($D\Psi_- = 0$) sectors with:

$$P_L = -\frac{1}{8\Box} D^\alpha \bar{D}^2 D_\alpha \tag{3.3}$$

$$P_+ = \frac{1}{16}\frac{\bar{D}^2 D^2}{\Box} \qquad P_- = \frac{1}{16}\frac{D^2 \bar{D}^2}{\Box} \tag{3.4}$$

$$P_C \equiv P_+ + P_- \, . \tag{3.5}$$

P_C projects out the real chiral part ($= \Phi + \bar{\Phi}$ for chiral Φ) of a real superfield and will be very handy in what follows. For a gauge field in a real scalar superfield V_R, the physical field strength involves only $P_L V_R$ and the gauge degrees of freedom are purely $P_C V_R$.

The graviton and gravitino belong to a supergravity or superspin-$\frac{3}{2}$ multiplet. The smallest field containing such a supergravity multiplet is a real vector superfield H_m. Naturally, this is the metric superfield which appears in superfield formulations of supergravity. It has superspin components

$$H_m = \frac{3}{2} \oplus 1 \oplus \frac{1}{2} \oplus \frac{1}{2} \oplus 0 \, . \tag{3.6}$$

We can isolate these components with a set of non-local projection operators:

$$\Pi^0_{mn} \equiv \omega_{mn} P_C \tag{3.7}$$
$$\Pi^{1/2}_{mn} \equiv \omega_{mn} P_L \tag{3.8}$$
$$\Pi^{3/2}_{mn} \equiv -\frac{1}{48}\sigma_m^{\alpha\dot\alpha}[D_\alpha,\bar D_{\dot\alpha}][D_\beta,\bar D_{\dot\beta}]\sigma_n^{\beta\dot\beta} - \frac{1}{8}D^\alpha \bar D^2 D_\alpha \delta_{mn} - \omega_{mn} + \frac{2}{3}\Pi^0_{mn} \tag{3.9}$$
$$\Pi^{1/2(T)}_{mn} \equiv \frac{1}{48}\sigma_m^{\alpha\dot\alpha}[D_\alpha,\bar D_{\dot\alpha}][D_\beta,\bar D_{\dot\beta}]\sigma_n^{\beta\dot\beta} - \Pi^0_{mn} \tag{3.10}$$
$$\Pi^1_{mn} \equiv \delta_{mn} - \Pi^0_{mn} - \Pi^{1/2(L)}_{mn} - \Pi^{1/2(T)}_{mn} - \Pi^{3/2}_{mn}\,. \tag{3.11}$$

These are defined for conciseness in terms of the vector projector ω_{mn} (3.1) and the scalar superfield projectors (3.3)–(3.5). We will not make use of Π^1_{mn} or $\Pi^{1/2(T)}_{mn}$ and include them only for completeness.

3.1 Massive supergravity

Now we would like to choose our lagrangian, following (3.2) as simply

$$\mathcal{L} = H_m \Box \Pi^{3/2}_{mn} H_n + m^2 H_m^2\,. \tag{3.12}$$

But this lagrangian, in contrast to (3.2), is non-local. To make it local, we must include another projector. The simplest choice is Π^0 which leads to:

$$\mathcal{L}_0 = -H_m \Box \Pi^{3/2}_{mn} H_n + \frac{2}{3} H_m \Box \Pi^0_{mn} H_n + m^2 H_m^2 \tag{3.13}$$
$$= \frac{1}{48} H_m \sigma_m^{\alpha\dot\alpha}[D_\alpha,\bar D_{\dot\alpha}][D_\beta,\bar D_{\dot\beta}]\sigma_n^{\beta\dot\beta} H_n + \frac{1}{8} H_m D^\alpha \bar D^2 D_\alpha H_m - (\partial H)^2 + m^2 H_m^2\,. \tag{3.14}$$

While this lagrangian is local, (3.13) shows that an additional superspin-0 degree of freedom $(\Pi^0 H)_m$ propagates, but with the opposite sign kinetic term from $(\Pi^{3/2} H)_m$. So one must be a ghost. The resolution is to introduce a new auxiliary chiral field Φ whose equations of motion force $\Phi = (\Pi^0 H)_m = 0$. This will be the chiral compensator. If we rewrote the Fierz-Pauli lagrangian in projector language, we would see that the metric trace h serves an analogous purpose: its equations of motion force $h = h_{mn,mn} = 0$ [22].

We can isolate the ghost $(\Pi^0 H)_m$ by defining:

$$H \equiv \partial_m H_m = H_C + H_L\,, \tag{3.15}$$

where $H_C = P_C H$ is a real chiral field, containing superspin-0, and $H_L = P_L H$ is linear and contains superspin-1. Then the sector of $\mathcal{L}_0$ involving H_C is:

$$\mathcal{L}_C = -\frac{2}{3} H_C^2 - H_C \frac{m^2}{\Box} H_C\,. \tag{3.16}$$

Thus, we introduce a new real chiral field Ψ_C and expand (3.16) to

$$\mathcal{L}_C = -\frac{2}{3}(H_C - \Psi_C)^2 - (H_C - \Psi_C)\frac{m^2}{\Box}(H_C + \Psi_C)\,. \tag{3.17}$$

It is then straightforward to see that the equations of motion for $H_C + \Psi_C$ and $H_C - \Psi_C$ force $H_C = \Psi_C = 0$.

To represent (3.17) as a local lagrangian note that any real chiral field can be written as $\Psi_C = \frac{3}{2}i(\Sigma - \bar{\Sigma})$, with Σ chiral. And any chiral field can be written as $\Sigma = -\frac{1}{4}\bar{D}^2 P$ for real P. Thus,

$$\Psi_C = \frac{3}{2}i(\Sigma - \bar{\Sigma}) = -\frac{3}{8}i(\bar{D}^2 P - D^2 P)\,. \tag{3.18}$$

Then we have, using (3.4) and integrating by parts:

$$\int d^8 z \Psi_C \frac{1}{\Box}\Psi_C = -\frac{9}{32}\int d^8 z D^2 P \frac{1}{\Box}\bar{D}^2 P = \frac{9}{4}\int d^8 z P^2\,. \tag{3.19}$$

So,

$$\mathcal{L}_C = -\frac{2}{3}(H_C - \frac{3}{2}i(\Sigma - \bar{\Sigma}))^2 - H_C\frac{m^2}{\Box}H_C + \frac{9}{4}m^2 P^2\,. \tag{3.20}$$

The complete quadratic lagrangian is now local:

$$\begin{aligned}\mathcal{L} &= -H_m\Box(\Pi^{3/2}H)_m + \frac{2}{3}H_m\Box(\Pi^0 H)_m + m^2 H_m^2 + 2iH_C(\Sigma - \bar{\Sigma}) - 3\bar{\Sigma}\Sigma + \frac{9}{4}m^2 P^2 \\ &= \frac{1}{48}H^m\sigma_m^{\alpha\dot{\alpha}}[D_\alpha, \bar{D}_{\dot{\alpha}}][D_\beta, \bar{D}_{\dot{\beta}}]\sigma_n^{\beta\dot{\beta}}H_n + \frac{1}{8}H_m D^\alpha\bar{D}^2 D_\alpha H_m - (\partial H)^2 + \\ &\quad + 2i(\Sigma - \bar{\Sigma})(\partial H) - 3\bar{\Sigma}\Sigma + m^2 H_m^2 + \frac{9}{4}m^2 P^2\,.\end{aligned} \tag{3.21}$$

In the massless limit, this is the linearized version of the old-minimal supergravity lagrangian with the chiral compensator [17].

We can check the lagrangian by introducing goldstones. At the linearized level, there is a set of longitudinal gauge transformations which correspond to

$$H_m \to H_m + \partial_m G_R \tag{3.22}$$

for a real superfield G_R. Thus, $H = (\partial H) \to H + \Box G_R$. And, because the massless part of the lagrangian is invariant under this gauge transformation, we can read off the transformation of Ψ_C from the first term in (3.17): $\Psi_C \to \Psi_C + \Box G_C$. Then $G_R = G_C + G_L$ comes in as

$$\mathcal{L}_C \supset -(H - \Psi_C)\frac{m^2}{\Box}(H + \Psi_C) \tag{3.23}$$
$$\to -m^2 G_L\Box G_L - 2m^2(H_C - \Psi_C)G_C + \cdots\,. \tag{3.24}$$

This is exactly what we should expect. The linear part of G_R, G_L, is a real multiplet with gauge invariance, so it contains the vector longitudinal modes. These get the correct kinetic term:

$$G_L\Box G_L = G_L D^\alpha\bar{D}^2 D_\alpha G_L\,. \tag{3.25}$$

The real chiral part of G_R, G_C, contains the scalar longitudinal mode of the graviton. It gets a kinetic term from mixing with $H_C - \Psi_C$. So the whole goldstone formalism for gravity, developed in [12] goes over beautifully to the supersymmetric case.

We will reproduce this argument more carefully in section 5 after we have proved the correctness of the linearized theory at the component level, and reviewed the non-linear transformations of supergravity. But first, we will comment on some alternatives to (3.21).

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3.2 Alternative free lagrangians

In a recent paper [1], two candidate (free) lagrangians for massive gravity were suggested, neither of which match (3.21). Since these lagrangians were only presented at the linearized level, it is unclear how to add interactions in a consistent way. In this section, we rederive them and explain why they are not promising candidates for a non-linear theory.

We observed above that the simple lagrangian involving only the superspin-$\frac{3}{2}$ projector is non-local, so we must include other superspin components, for example Π^0 or Π^1. Let us allow for an arbitrary linear combination of these two projectors. Then the general local lagrangian is:

$$\begin{aligned}
\mathcal{L}_H &= -H_m \Box \Pi^{3/2}_{mn} H_n + \alpha \frac{2}{3} H_m \Box \Pi^0_{mn} H_n + (1-\alpha)\frac{2}{3} H_m \Box \Pi^{1/2}_{mn} H_n + m^2 H_m^2 &(3.26)\\
&= L_{3/2} + (1-\alpha)\frac{2}{3} H_L H_L - \alpha \frac{2}{3} H_C H_C - H \frac{m^2}{\Box} H\,, &(3.27)
\end{aligned}$$

where $H = \partial_m H_m = H_L + H_C$ are the same objects from section 3.1. Now we want to introduce a real auxiliary superfield Ψ_R so that the equations of motion enforce $H = \Psi_R = 0$. Then the most general local lagrangian involving H and Ψ_R is:

$$\begin{aligned}
\mathcal{L}_{H\Psi} = {}& \frac{2(1-\alpha)}{3} H P_L H - \frac{2\alpha}{3} H P_C H - \frac{m^2}{\Box} H^2 + \frac{4}{3}\Psi_R H + \\
& + \frac{1}{2}\beta_1 \Psi_R \Box P_L \Psi_R + \frac{1}{2}\beta_2 \Psi_R \Box P_C \Psi_R + \frac{1}{2}\gamma {\Psi_R}^2\,.
\end{aligned}$$

Substituting the equation of motion for Ψ_R into that of H gives:

$$(\beta_1 \Box P_L + \beta_2 \Box P_C + \gamma)\left(\frac{4(1-\alpha)}{3}\Box P_L - \frac{4\alpha}{3}\Box P_C - 2m^2\right) H = \frac{16}{9}\Box H\,. \tag{3.28}$$

Since $P_L P_C = P_C P_L = 0$ and $P_C + P_L = 1$ there are two distinct solutions which force $H = \Psi_R = 0$. The first has $\alpha = 0$ and so only involves the Π^1 projector:

$$\begin{aligned}
\mathcal{L}^{\alpha=0}_{H\Psi} &= \frac{2}{3}(H_L + \Psi_L)^2 - \frac{m^2}{\Box} H_L^2 - \frac{1}{\Box}\left(m H_C - \frac{2}{3m}\Box \Psi_C\right)^2 + \frac{2}{3}\Psi_C^2 &(3.29)\\
&= \frac{1}{48} H_m \sigma^{\alpha\dot\alpha}_m [D_\alpha, \bar D_{\dot\alpha}][D_\beta, \bar D_{\dot\beta}] \sigma^{\beta\dot\beta}_n H_n + \frac{1}{8} H_m D^\alpha \bar D^2 D_\alpha H_m - \frac{1}{3}(\partial_m H_m)^2 + m^2 H_m^2 + \\
&\quad + \frac{4}{3}\Psi_R \partial_m H^m + \frac{2}{3}\Psi_R^2 - \Psi_R \frac{\{D^2, \bar D^2\}}{36 m^2}\Psi_R\,. &(3.30)
\end{aligned}$$

The second has $\alpha = 1$ and only involves the Π^0 projector:

$$\begin{aligned}
\mathcal{L}^{\alpha=1}_{H\Psi} &= -\frac{2}{3}(H_C - \Psi_C)^2 - \frac{m^2}{\Box} H_C^2 - \frac{1}{\Box}\left(m H_L - \frac{2}{3m}\Box \Psi_L\right)^2 - \frac{2}{3}\Psi_L^2 &(3.31)\\
&= \frac{1}{48} H_m \sigma^{\alpha\dot\alpha}_m [D_\alpha, \bar D_{\dot\alpha}][D_\beta, \bar D_{\dot\beta}] \sigma^{\beta\dot\beta}_n H_n + \frac{1}{8} H_m D^\alpha \bar D^2 D_\alpha H_m - (\partial_m H_m)^2 + m^2 H_m^2 + \\
&\quad + \frac{4}{3}\Psi_R \partial_m H^m - \frac{2}{3}\Psi_R^2 - \Psi_R \frac{D^\alpha \bar D^2 D_\alpha}{18 m^2}\Psi_R\,. &(3.32)
\end{aligned}$$

These are the two solutions given in [1].

Let us consider these two solutions in turn. For the first solution, $\alpha = 0$, if we take $m \to 0$ then Ψ_C decouples. We are left with a linearized supergravity theory with a vector auxiliary field. As discussed in [1] and [21] there is no known interacting theory of supergravity with this auxiliary multiplet structure. It is similar to new-minimal supergravity, and equivalent at the linearized level. But we do not know how to add interactions in a consistent way and we cannot approach the non-linear theory without solving the much harder problem of generating a new formulation of supergravity.

For the $\alpha = 1$ theory, the $m \to 0$ limit decouples Ψ_L and we are left with old-minimal supergravity with the chiral compensator set to zero. So in this case, we can in principle consider an interacting theory. However, the component form has a bosonic sector which is essentially equivalent to (2.3). There is a propagating vector field which simply does not decouple. In superfield terms, we have a real field Ψ_R which is not related in any way to the auxiliary fields of the massless sector.

4. Components

To show that the formal procedure of the previous section is mathematically sound, and to get a more practical understanding of the lagrangian, we will now work out the components of equation (3.21). We use the notation $|$ to indicate evaluation at $\theta = \bar\theta = 0$.

For the metric superfield we define

$$\begin{aligned}
&H^n| = A^n \quad -\frac{1}{4}D^2 H^n| = F^n \quad -\frac{1}{4}\sigma_m^{\alpha\dot\alpha}[D_\alpha, \bar D_{\dot\alpha}]H^n| = V_m^n \\
&\frac{1}{32}\{D^2, \bar D^2\}H^n| = D^n + \frac{1}{2}\Box A^n \\
&\frac{i}{16}\bar\sigma^{m\dot\beta\beta}\bar D^2 D_\beta H_{\alpha\dot\beta}| = \psi_\alpha^m \quad D_\alpha H_m| = \chi_{\alpha m}
\end{aligned} \tag{4.1}$$

and pull off the symmetric, antisymmetric and trace parts of V_m^n

$$V_{nm} = v_{mn} + \omega_{mn} + \frac{1}{4}\eta_{mn} h \tag{4.2}$$

$$v_{mn} - v_{nm} = \omega_{mn} + \omega_{nm} = v_{mm} = 0\,. \tag{4.3}$$

For the prepotential of the chiral compensator, we define

$$P| = p \quad -\frac{1}{4}\bar D^2 P| = \frac{1}{2}(s + it) \quad -\frac{1}{4}\sigma_m^{\alpha\dot\alpha}[D_\alpha, \bar D_{\dot\alpha}]P| = b_m \quad \frac{1}{32}\{D^2, \bar D^2\}P| = d \tag{4.4}$$

$$D_\alpha P| = i\lambda_\alpha \quad -\frac{1}{4}D_\alpha \bar D^2 P| = \zeta_\alpha\,. \tag{4.5}$$

Then, after an invigorating calculation, the linearized lagrangian (3.21) turns into:

$$\begin{aligned}
\mathcal{L} = &\frac{1}{2}v_{mn}\Box v_{mn} + (\partial_m v_{mn})^2 - \frac{1}{6}(h - 6s)(\partial\partial v) - \frac{1}{48}(h - 6s)\Box(h - 6s) + &(4.6)\\
&+\frac{4}{3}(D_m + \frac{1}{4}\Omega_m - \frac{3}{4}t_{,m} + \frac{1}{2}\partial A_{,m})^2 - \frac{4}{3}|(\partial F) + \frac{3}{4}(\partial b) - \frac{3}{2}id|^2 &(4.7)\\
&+m^2\Big[-\frac{1}{2}v_{mn}^2 - \frac{1}{2}\omega_{mn}^2 - \frac{1}{8}h^2 + \frac{9}{8}s^2 + \frac{9}{8}t^2 + \frac{1}{2}A_m\Box A_m + 2D\cdot A + 2|F_m|^2 + &(4.8)
\end{aligned}$$

$$+\frac{9}{2}pd-\frac{9}{8}p\Box\bar{p}-\frac{9}{8}b_m^2\Big]- \tag{4.9}$$

$$-2i\bar{\psi}_n(\bar{\sigma}_p\sigma_m\bar{\sigma}_n)\partial_m\psi_p-\left(\frac{4}{3}i\psi_n\sigma_m\partial_n\bar{\psi}_m+\text{h.c}\right)+$$

$$+\left(-\frac{2}{3}i\partial_n\bar{\chi}_n\bar{\sigma}_m\sigma_p\partial_p\bar{\psi}_m+\text{h.c.}\right)- \tag{4.10}$$

$$-\frac{2}{3}i\partial_p\partial_n\bar{\chi}_n\bar{\sigma}_p\partial_m\chi_m+(i\partial_n\bar{\chi}_n\sigma_m\partial_m\zeta+\text{h.c.})+i\frac{3}{2}\bar{\zeta}\sigma_m\partial_m\zeta+$$

$$+\left(\frac{1}{2}i\partial_\rho\psi_n\sigma_p\bar{\sigma}_n\zeta+\text{h.c.}\right)+ \tag{4.11}$$

$$+m^2\left[-(i\chi_m\sigma_n\bar{\sigma}_m\psi_n+h.c)+\frac{9i}{4}\lambda\sigma^m\partial_m\bar{\lambda}-i\chi_m\sigma_n\partial_n\bar{\chi}_m-\frac{9i}{4}\lambda\zeta+\frac{9i}{4}\bar{\lambda}\bar{\zeta}\right], \tag{4.12}$$

where $\Omega_m=\varepsilon_{mnpq}\partial_n\omega_{pq}$.

Notice the convenient grouping in (4.6) and (4.7). The linearized gauge invariance of the massless sector can practically be read off the lagrangian. If not for the mass term, we could use this symmetry to go to a gauge where $\omega=A=F=\chi=t=s=\lambda=\zeta=0$. However, we have broken the gauge invariance, and the best we can do is perform field redefinitions:

$$h\to-3h+6s \tag{4.13}$$

$$d\to d+\frac{1}{3}(i\partial F-i\partial\bar{F}) \tag{4.14}$$

$$b_m\to 2b_m-\frac{2}{3}(F_m+\bar{F}_m) \tag{4.15}$$

$$F_m\to F_m-\frac{3}{4}ip_{,m} \tag{4.16}$$

$$t\to t-\frac{2}{3}\partial A \tag{4.17}$$

$$D_m\to D_m-\frac{1}{4}\Omega_m-\frac{3}{4}t_{,m} \tag{4.18}$$

$$\psi_p\to\frac{i}{2\sqrt{2}}(\psi_p^R-\frac{1}{2}\sigma_p\bar{\sigma}_q\psi_q^R)-\frac{3}{4}i\sigma_p\bar{\zeta}-\frac{1}{2}\sigma_p\partial_q\bar{\chi}_q\,. \tag{4.19}$$

Thus, using the conventional $h_{mn}=v_{mn}+\frac{1}{4}\eta_{mn}h$, replacing $\chi_q=\frac{1}{\sqrt{2}}\psi_q^L$, and rescaling s, b_m, λ, d and ζ by a factor of $\frac{2}{3}$ each, we have:

$$\mathcal{L}=\frac{1}{2}h_{mn}\Box h_{mn}+h_{mn,m}^2+h(\partial\partial h)-\frac{1}{2}h\Box h-\frac{4}{3}d^2-\frac{4}{3}(\partial b)^2+\frac{4}{3}D_m^2- \tag{4.20}$$

$$-\frac{1}{2}\varepsilon^{mnpq}\psi_m^R\sigma_n\partial_p\bar{\psi}_q^R+ \tag{4.21}$$

$$+m^2\Big[-\frac{1}{2}h_{mn}^2-h^2+3hs-\frac{3}{2}s^2+3dp-2b_m^2-\frac{1}{2}(F_m-\bar{F}_m)^2-\frac{1}{2}b_m(F_m+\bar{F}_m)+ \tag{4.22}$$

$$+\frac{1}{2}t^2-\frac{1}{2}\omega_{mn}^2+\frac{1}{2}A_m\Box A_m+2D_mA_m-\frac{1}{2}\Omega_mA_m+\frac{1}{2}(\partial A)^2+ \tag{4.23}$$

$$+\frac{1}{4}\psi_m^L[\sigma_n,\bar{\sigma}_m]\psi_n^R+\text{h.c.}+i\lambda\sigma_m\partial_m\bar{\lambda}-i\lambda\zeta+i\bar{\lambda}\bar{\zeta}+ \tag{4.24}$$

$$+\frac{1}{\sqrt{2}}\left(\zeta\sigma_m\psi_m^L+\text{h.c.}\right)-\frac{i}{4}\psi_m^L(\sigma_m\bar{\sigma}_n\sigma_p+\sigma_p\bar{\sigma}_n\sigma_m)\partial_n\bar{\psi}_p^L\Big]. \tag{4.25}$$

The massless sector of this lagrangian now looks like supergravity in Wess-Zumino gauge. There is the correct kinetic term for the spin-2 and spin-$\frac{3}{2}$ fields and the auxiliary vector and complex scalar forming the off-shell supergravity multiplet. In this form, it is not hard to integrate out the auxiliary fields, which leads to the on-shell quadratic lagrangian

$$\begin{aligned} L = &\frac{1}{2}h_{mn}\Box h_{mn} + h^2_{mn,m} + h(\partial\partial h) - \frac{1}{2}h\Box h - \frac{1}{2}m^2(h^2_{mn} - h^2) - \frac{1}{2}\varepsilon^{mnpq}\psi^R_m\sigma_n\partial_p\bar{\psi}^R_q - \\ &- \frac{3}{2}m^2\left[\frac{1}{4}(A_{m,n} - A_{n,m})^2 + \frac{1}{2}m^2A^2_m\right] + m^2\left[-\frac{1}{2}\varepsilon^{mnpq}\psi^L_m\sigma_n\partial_p\bar{\psi}^L_q + \frac{1}{4}\psi^L_m[\sigma_n, \sigma_m]\psi^R_n\right]. \end{aligned} \tag{4.26}$$

After rescaling, this is (1.2).

The field redefinitions that we made, like the gauge choice discussed above, eliminate the dependence on $\omega, A, F, \chi, t, s, \lambda$ and ζ in the massless sector. This follows, of course, from the fact that gauge transformations *are* field redefinitions. But it also illustrates that if we perform, as a field redefinition, the non-linear transformation which puts the fields in a particular gauge, it will produce interactions in the massive sector of the lagrangian. As we will show in the next two sections, it is much easier to study the interactions by introducing goldstones directly in terms of superfields. But it is also useful to see, at least schematically, the fields responsible for strong coupling by introducing component goldstones into the on-shell lagrangian (4.26).

We need vector and scalar goldstones for h_{mn}, a scalar goldstone for A_m, and goldstinos for ψ^R and ψ^L

$$h_{mn,n} \to h_{mn,n} + \pi^h_{m,n} + \phi^h_{,m,n} \tag{4.27}$$

$$A_m \to A_m + \phi^A_{,m} \tag{4.28}$$

$$\psi^{R,L}_m \to \psi^{R,L}_m + \chi^{R,L}_{,m}\,. \tag{4.29}$$

Then, putting back the $M^2_{\rm Pl}$ multiplying (4.26), the kinetic terms are schematically

$$\begin{aligned} L \to\ & M^2_{\rm Pl}\left\{h\partial^2 h + \bar{\psi}^R\partial\psi^R\right\} + \\ & + M^2_{\rm Pl}m^2\left\{A\Box A + \bar{\psi}^L\partial\psi^L + h\partial^2\phi^h + \bar{\psi}^L\partial\chi^R + \bar{\psi}^R\partial\chi^L + \pi^h\partial^2\pi^h\right\} + M^2_{\rm Pl}m^4\phi^A\Box\phi^A\,. \end{aligned} \tag{4.30}$$

From this we can read off the canonical normalizations. For example, $h_c = M_{\rm Pl}h$ and then because of the kinetic mixing, $\phi^h_c = M_{\rm Pl}m^2\phi^h$. We find that ϕ^h, ϕ^A, and χ^L all get factors of $M_{\rm Pl}m^2$ in their normalization. This is perfectly consistent with supersymmetry, as they fall into a chiral multiplet. If we were to work out the higher order terms in the on-shell lagrangian, they would include interactions of ϕ^h, ϕ^A, and χ^L with the characteristic scale $\Lambda_5 \equiv (M_{\rm Pl}m^4)^{1/5}$. But as we have mentioned, it is significantly easier to work with superfields, as we will see in section 6.

5. Non-linear theory

To study the interacting theory, we proceed in a way similar to [12]. The idea is to restore local supersymmetry to the action (1.1) by introducing goldstone superfields. By the goldstone equivalence theorem, these represent the longitudinal degrees of freedom which become strongly coupled at high energy. To introduce the goldstones, we will perform a finite supergravity transformation on (1.1), parameterized by a superfield L_α. Because (1.1) is not invariant, the lagrangian will then depend on L_α. If we now interpret L_α as a field, it contains the goldstones. The new lagrangian, with L_α, will be invariant, because we can absorb an additional transformation in a shift of L_α. And so the restored symmetry keeps the original supergravity fields transverse, and the strongly coupled longitudinal modes appear in the goldstone fields. In order to carry out this procedure, we need the finite supergravity transformations. In this section, we attempt to motivate and review the formalism we have found most convenient [23] (see also [17, 18, 19]), and we work out the transformations to second order. The interactions will be studied using the goldstones in section 6.

The natural starting point for non-linear supergravity is to consider local translations of the form

$$\Psi(x^m, \theta, \bar\theta) \to \Psi(x^m + \delta x^m, \theta + \delta\theta, \bar\theta + \bar{\delta\theta}), \tag{5.1}$$

with δx^m real and $\delta\theta$ a complex Weyl fermion. This is the natural generalization of general coordinate transformations to real superspace. For constant $\delta x^m = i\theta\sigma^m\delta\bar\theta - i\delta\theta\sigma^m\bar\theta$ it is just a global supersymmetry rotation.

However, as we will justify *a posteriori*, it is convenient to go further and consider local translations of *complex* superspace

$$z^M = (z^m, \theta, \bar\theta) \to (z^m + \delta z^m, \theta + \delta\theta, \bar\theta + \delta\bar\theta), \tag{5.2}$$

where now $\delta\theta \neq \overline{(\delta\bar\theta)}$. This group can be represented by

$$z \to z' = (e^\Lambda z), \tag{5.3}$$

where Λ is a complex supervector field:

$$\Lambda = \Lambda^M \partial_M = \Lambda^m \partial_m + M^\alpha D_\alpha + \bar N_{\dot\alpha} \bar D^{\dot\alpha}. \tag{5.4}$$

While $\delta z^M = \Lambda^M$ holds only to first order, (5.4) is nevertheless a faithful representation of (5.2) (more precisely, of *contractible* transformations, connected to the identity). Keep in mind that in our conventions $\Lambda^m\partial_m = -\frac{1}{2}\Lambda^{\alpha\dot\alpha}\partial_{\alpha\dot\alpha}$.

Fields transform as

$$\Psi(z) \to \Psi'(z) = e^\Lambda \Psi(z). \tag{5.5}$$

Note that $(\Psi_1\Psi_2)' = \Psi_1'\Psi_2'$ because of the identity $(e^\Lambda\Psi) = e^\Lambda \Psi e^{-\Lambda}$. Also note that we normally deal with superfields $\Psi(x)$ which depend only on *real* superspace. In that case, one can still define an active transformation by $\Psi(x) \to e^\Lambda\Psi(x)$ but can no longer interpret it as a passive coordinate shift.

As it is, with Λ^M arbitrary superfields, this supergeneral coordinate transformation (SUGC) group is too large. We will now restrict it. First, we want chiral fields to remain chiral. Since

$$\Phi \to e^\Lambda \Phi = \Phi + \Lambda\Phi + \cdots, \tag{5.6}$$

we need

$$[\bar{D}, \Lambda]\Phi = 0\,. \tag{5.7}$$

This produces constraints on M^α and Λ^m which are solved by

$$\Lambda_{\alpha\dot{\alpha}} = -2i\bar{D}_{\dot{\alpha}}L_\alpha \quad M_\alpha = -\frac{1}{4}\bar{D}^2 L_\alpha\,, \tag{5.8}$$

for an unconstrained spinor valued superfield L_α. At this point N_α is still unconstrained.

The transformation $\Psi \to e^\Lambda\Psi$ is for general superfields, so if we want antichiral fields to remain antichiral we would need

$$[D, \Lambda]\bar{\Phi} = 0\,. \tag{5.9}$$

To satisfy both (5.7) and (5.9) would restrict the group too much (down to non-susy GC). So we need some other way of keeping antichiral fields antichiral. Enter the metric. We take the metric H to be a real supervector field

$$H = H^m\partial_m + H^\alpha D_\alpha + \bar{H}_{\dot{\alpha}}\bar{D}^{\dot{\alpha}}\,, \tag{5.10}$$

with $H^m = \bar{H}^m$. We make H transform as

$$e^{2iH} \to e^{2iH'} = e^{\bar{\Lambda}}e^{2iH}e^{-\Lambda}\,. \tag{5.11}$$

Now, we can define a covariantly antichiral field by

$$\bar{\Phi}^\ddagger \equiv e^{2iH}\bar{\Phi} \to e^{\bar{\Lambda}}\bar{\Phi}^\ddagger\,. \tag{5.12}$$

Since (5.7) implies $[D, \bar{\Lambda}]\bar{\Phi} = 0$, the antichirality of $\bar{\Phi}^\ddagger$ is preserved.

Perturbatively,

$$H \to H - \frac{i}{2}(\bar{\Lambda} - \Lambda) + [\Lambda + \bar{\Lambda}, H] - \frac{i}{2}[\Lambda, \bar{\Lambda}] + \cdots\,. \tag{5.13}$$

So,

$$H^\alpha \to H^\alpha + \frac{i}{2}(M^\alpha - N^\alpha)\,. \tag{5.14}$$

Since N^α remains arbitrary, we can use it to set

$$H^\alpha = 0\,. \tag{5.15}$$

Thus, $N_\alpha = M_\alpha$ to first order, and the allowed SUGC transformations are determined completely in terms of L_α.

Although we will not need it here, for clarity and completeness, the gauge condition to next order is:

$$N^\alpha = M^\alpha - H^m(\partial_m M^\alpha) - \frac{i}{2}(\bar{\Lambda}^m - \Lambda^m)(i\partial_m M^\alpha) + \cdots\,. \tag{5.16}$$

In contrast, the other gauge constraint (5.7), $[\bar{D}, \Lambda]\Phi = 0$, implies $\bar{D}e^\Lambda\Phi = 0$ so (5.8) does not get further corrections.

In summary, with the conditions (5.7) and (5.15), the group of transformations $z \to (e^{\Lambda} z)$ we want are determined completely by a spinor superfield L_α:

$$\Lambda = (i\bar{D}_{\dot\alpha} L_\alpha)\partial_{\alpha\dot\alpha} + \left(-\frac{1}{4}\bar{D}^2 L^\alpha\right) D_\alpha + \left(-\frac{1}{4}D^2 \bar{L}_{\dot\alpha} + \cdots\right) \bar{D}^{\dot\alpha} . \tag{5.17}$$

This group is a subgroup of complex SUGC representing superconformal transformations. At the linearized level, it is also the gauge invariance associated to a massive superspin-$\frac{3}{2}$ multiplet. As we discussed in section 3, it is impossible to write down a local quadratic lagrangian involving only superspin-$\frac{3}{2}$. There, the chiral compensator came in as a Lagrange multiplier forcing the other spins to vanish on shell. In the massless theory, one can use up some of the superconformal invariance to set the compensator to zero. But the essential relation between superconformal transformations and the superspin-$\frac{3}{2}$ field of supergravity remains. From now on, we will take SUGC to mean just these symmetries, (5.17).

The non-linear generalization of constructing quadratic lagrangians with linear invariance is constructing invariant integrals. As in the non-supsersymmetric case, the integral of a scalar is not invariant, and so we need to introduce scalar densities, the analog of $\sqrt{g}$. But, in supergravity, it is difficult to construct a density out the metric H^m and so we simply introduce the chiral compensator for this purpose. The transformation of a supersymmetric scalar density, to first order, is described by right action of a supervector field,

$$\Psi \overleftarrow{\Lambda} \equiv (-1)^{\varepsilon_M} \partial_M (\Lambda^M \Psi) = \partial_m (\Lambda^m \Psi) - D^\alpha (M_\alpha \Psi) - \bar{D}_{\dot\alpha} (\bar{N}^{\dot\alpha} \Psi) . \tag{5.18}$$

This is a total derivative, and can be used to construct invariant Kahler potentials. If we only integrate over $d^2\theta$ the $\bar{D}$ term is not invariant and we must use

$$\Lambda_{\mathrm{CH}} \equiv \Lambda^m \partial_m + M^\alpha D_\alpha . \tag{5.19}$$

So, for the non-linear theory, we define the chiral compensator $e^{3\Sigma}$ as a chiral density with the transformation law

$$e^{3\Sigma} \to e^{3\Sigma} e^{\overleftarrow{\Lambda}_{\mathrm{CH}}} . \tag{5.20}$$

Note that for a chiral field $\Lambda\Phi = \Lambda_{\mathrm{CH}}\Phi$. This lets us construct invariant superpotentials. Indeed, one can check that for a chiral superpotential W,

$$(e^{3\Sigma} e^{\overleftarrow{\Lambda}_{\mathrm{CH}}})(e^{\Lambda} W) = (e^{3\Sigma} W) e^{\overleftarrow{\Lambda}_{\mathrm{CH}}} , \tag{5.21}$$

and so $\int d^2\theta e^{3\Sigma} W$ is invariant. There is also a way to construct Kahler potentials which are invariant under finite transformations, involving both H and $e^{3\Sigma}$, but it is more complicated and we will not present it here.

Expanding to second order, the transformations are:

$$\begin{aligned}
\delta H_{\alpha\dot\alpha} &= \bar{D}_{\dot\alpha} L_\alpha + i\bar{D}^{\dot\beta} L^\beta \partial_{\beta\dot\beta} H_{\alpha\dot\alpha} - \frac{1}{4}(\bar{D}^2 L^\beta) D_\beta H_{\alpha\dot\alpha} - iH^{\beta\dot\beta} \partial_{\beta\dot\beta} \bar{D}_{\dot\alpha} L_\alpha - \\
&\quad -i\sigma^{m\dot\beta\beta} (\bar{D}_{\dot\beta} L_\beta) \partial_m D_\alpha \bar{L}_{\dot\alpha} - \frac{1}{4}(\bar{D}^2 L^\beta) D_\beta (\bar{D}_{\dot\alpha} L_\alpha - D_\alpha \bar{L}_{\dot\alpha}) + \text{h.c.} \qquad (5.22) \\
\delta\Sigma &= \frac{1}{12}\bar{D}^2 D_\alpha L^\alpha + \frac{1}{96}\bar{D}^2 (L^\alpha D_\alpha \bar{D}^2 D_\beta L^\beta) - \frac{1}{4}\bar{D}^2 (L^\alpha D_\alpha \Sigma) \qquad (5.23) \\
\delta P &= -\frac{1}{3} D_\alpha L^\alpha - \frac{1}{24}(L^\alpha D_\alpha \bar{D}^2 D_\beta L^\beta) + (L^\alpha D_\alpha \Sigma) + \text{h.c.} \qquad (5.24)
\end{aligned}$$

6. Interacting Goldstone superfields

To study the interacting theory, we will now introduce goldstones. Because we have the transformation laws entirely in terms of superfields, we can study the interactions in a manifestly supersymmetric way. All of the goldstones fit into the spinor-valued real superfield L_α. The strong interactions involve the longitudinal modes of the fields in this multiplet, which we can isolate by writing $L_\alpha = D_\alpha G_R$ for real G_R. And the scalar longitudinal modes are contained in the real chiral part of G_R, which we can isolate with $G_R \to G_R + G + \bar{G}$. This G, without a subscript, is a chiral field. Therefore, we project out the strongest modes by setting:

$$L_\alpha = \frac{i}{2} D_\alpha G \quad \bar{L}^{\dot{\alpha}} = -\frac{i}{2}\bar{D}^{\dot{\alpha}}\bar{G}, \qquad \bar{D}G = 0\,. \tag{6.1}$$

This leads to the very simple form for the linear gauge transformations (5.17)

$$\Lambda = -2i(\partial^m G)\partial_m\,. \tag{6.2}$$

These are coordinate transformations of the form $\delta x^m = \partial^m G$ which include the scalar longitudinal transformation, $\delta h_{mn} = G^h_{,m,n}$ leading to strong coupling in gravity. In fact, this chiral multiplet effectively contains the component goldstones ϕ^h, ϕ^A and χ^L which we began to analyze at the end of Section 4.

The second order transformations (5.22) and (5.24) are:

$$\begin{aligned}\delta H^m &= \partial^m(G+\bar{G}) - 2i\partial^n G\partial_n\partial_m\bar{G} + 2i\partial^n\bar{G}\partial_n\partial_m G - 2i\partial^n(G-\bar{G})\partial_n H^m + \\ &\quad + 2iH^n\partial_n\partial_m(G-\bar{G}) \\ \delta P &= \frac{i}{6}D^2 G + \frac{i}{2}D^\alpha G D_\alpha \bar{D}^2 P - \frac{1}{6}D^\alpha G\Box D_\alpha G + \text{h.c.}\,.\end{aligned} \tag{6.3}$$

So, at quadratic order, the mass terms become

$$m^2\int H_m^2 + \frac{9}{4}P^2 \to m^2\int (H_m + \partial^m(G+\bar{G}))^2 + \frac{9}{4}\left(P + \frac{i}{6}D^2G - \frac{i}{6}\bar{D}^2\bar{G}\right)^2\,. \tag{6.4}$$

After an integration by parts and an application of the identity $\bar{D}^2D^2G = 16\Box G$ this simplifies to

$$m^2\int H_m^2 + \frac{9}{4}P^2 - 2\left(\partial H - \frac{3i}{2}(\Sigma - \bar{\Sigma})\right)(G+\bar{G})\,. \tag{6.5}$$

As expected, the kinetic term for G has vanished, and G picks up a kinetic term from mixing. Moreover, it mixes with the same combination of metric and compensator that Σ itself does in the massless lagrangian, (3.21). So we can undo the mixing with a Weyl transformation $\Sigma \to \Sigma - iG$, producing an an ordinary $\bar{G}G$ kinetic term. This is exactly analogous to how the kinetic mixing works in non-supersymmetric massive gravity. Note that this Weyl transformation will produce a coupling of G to matter with $M_{\rm Pl}$ strength. So this coupling will remain even in the limit $m \to 0$. Therefore, there is a supersymmetric analog of the vDVZ discontinuity [10, 11].

To canonically normalize the fields, note that the metric and the compensator have kinetic terms in the massless action, which we have defined with a coefficient $M_{\rm Pl}^2$. Schematically,

$$\mathcal{L} = M_{\rm Pl}^2(H\partial^2 H + \bar{\Sigma}\Sigma) + M_{\rm Pl}^2 m^2(\partial H + \Sigma)G\,. \tag{6.6}$$

So $H_m^c = M_{\rm Pl}H_m$, $\Sigma^c = M_{\rm Pl}\Sigma$ and $G^c = M_{\rm Pl}m^2 G$. If we had included the linear part of the goldstone multiplet $G_R = G_L + G + \bar{G}$ we would see that it gets a kinetic term from the mass without mixing and is normalized by $G_L^c = M_{\rm Pl}mG_L$. But the chiral mode, G, has the smallest kinetic term and the largest interactions. It is not hard to see that strongest interactions come from terms cubic in G. Expanding the mass terms to next order order,we have:

$$\begin{aligned} &m^2 H_m^2 + \frac{9}{4}P^2 \to \\ &\quad\to \frac{-2i}{M_{\rm Pl}m^4}\left(-3\partial_a G_c^\dagger \Box G_c \partial_a G_c - 2\partial_a {G_c}^\dagger \Box G_c \partial_a G_c^\dagger + G_c \Box {G_c}^\dagger \Box {G_c}^\dagger\right) + \text{h.c.} + \cdots\,. \end{aligned}$$

These vertices lead to amplitudes which violate unitarity at the energy $(M_{\rm Pl}m^4)^{1/5} \equiv \Lambda_5$. So we take the cutoff to be $\Lambda \sim \Lambda_5$.

In a consistent effective field theory, naive dimensional analysis [24]–[26] tells us that we must include all higher dimension operators suppressed by the cutoff. In supersymmetric theories, the easiest way to keep the dimensions straight is to rescale θ to be dimensionless by $\theta \to \frac{\hat{\theta}}{\sqrt{\Lambda}}$. Then canonically normalized bosonic superfields, such as $\hat{\Psi}(x, \frac{\hat{\theta}}{\sqrt{\Lambda}}, \frac{\hat{\bar{\theta}}}{\sqrt{\Lambda}})$ have mass dimension 1. In our case, we know that because the goldstone field G is strongly coupled at Λ, we must include operators in the Kahler potential like

$$\frac{\Lambda^2}{16\pi^2}\left(\frac{\partial}{\Lambda}\right)^a\left(\frac{4\pi\hat{G}_c}{\Lambda}\right)^b = \frac{\Lambda^2}{16\pi^2}\left(\frac{\partial}{\Lambda}\right)^a\left(\frac{4\pi M_{\rm Pl}m^2\hat{G}}{\Lambda^3}\right)^b. \tag{6.7}$$

We have included a $(16\pi^2)^{-1}$ in front as a loop factor, and a 4π next to G for canonical normalization. Matching the $a = 6$, $b = 3$ term to the tree level expression enforces $\Lambda < (4\pi)^{1/5}\Lambda_5 \sim 1.6\,\Lambda_5$. Since we are neglecting factors of order 1, it is consistent to ignore the 4π's in the our analysis.

Now, G comes out of H, so the corresponding unitary gauge term is

$$\Lambda^2\left(\frac{\partial}{\Lambda}\right)^a\left(\frac{M_{\rm Pl}m^2}{\Lambda^3}\frac{\hat{H}_c}{M_{\rm Pl}}\right)^b = \Lambda^2\left(\frac{\partial}{\Lambda}\right)^a\left(\hat{e}\frac{\hat{H}_c}{\Lambda}\right)^b. \tag{6.8}$$

The important point is that the couplings of the transverse modes do not enter as $\hat{H}_c/\Lambda$ but are down by a weak coupling factor of

$$\hat{e} \equiv \frac{m^2}{\Lambda^2} = \left(\frac{m}{M_{\rm Pl}}\right)^{2/5}. \tag{6.9}$$

7. Dimensional deconstruction

Now that we understand the lagrangian for massive supergravity, we can string together an extra dimension. This works essentially the same way for supergravity as it does for gauge theories. To construct an N-site model, we begin with N independent 4D supergravity theories:

$$S = \int d^4x d^4\theta \sum_j M_j^2 \mathcal{E}^{-1}[H_j^m, \Sigma_j]\,. \tag{7.1}$$

M_j are the Planck scales on each site, which we may take to be distinct. Note that there is only one set of coordinates, but the lagrangian is invariant under N copies of SUGC. Now we add nearest-neighbor interactions which break SUGC^N down to the diagonal SUGC subgroup:

$$L_U = \sum_j M_j^2 \mathcal{E}^{-1}[H_j^m, \Sigma_j] + M_j^2 m_j^2 \left\{ (H_{j+1}^m - H_j^m)^2 + \frac{9}{4}(P_{j+1} - P_j)^2 + \cdots_j \right\}. \tag{7.2}$$

Here, m_j characterize the masses. If we set $M_j = M$ and $m_j = m$ then the theory contains a tower of massive supergravity multiplets with couplings $M_{\rm Pl}^2 = NM^2$ and masses $m_n = m \sin \frac{n\pi}{N}$, with $n = 0 \ldots N-1$. For large N this approximates one KK tower of a compactified 5D supergravity theory, as should be expected for a discretization. Also, $\cdots_j$ stands for the terms needed to make the mass terms invariant under the SUGC group on site j. This is done by the standard procedure for adding gravitational interactions (covariant derivatives and chiral compensator terms) to a flat space Kahler potential. The analog in ordinary gravity is to write $\sqrt{g} h_{\mu\nu}^2$ instead of simply $h_{\mu\nu}^2$ in the mass term. While we ascribe no meaning to the statement that these additional terms are "necessary," they are nevertheless helpful to guarantee that the low energy theory has diagonal SUGC symmetry which gives it a chance at being phenomenologically viable.

How does sequestering work in theory space? Suppose we have some chiral field Φ_1 on site 1 and another chiral field Φ_N on site N. Since Φ_1 and Φ_N are at different sites, we can plausibly omit contact terms like $\Phi_1^\dagger \Phi_1 \Phi_N^\dagger \Phi_N$ at tree level. But such terms will be generated by quantum corrections, and we must estimate their size to see how they are suppressed by the "distance" between sites 1 and N. Before doing that, it is instructive to restore the broken SUGC^N symmetry by promoting the lagrangian to a non-linear sigma model with the introduction of compensating goldstone superfields. As we saw in section 5, a concise parameterization of the symmetry group can be made with a spinor superfield L_α, so we include $N-1$ spinor superfields, one for each link. The non-linear sigma model is then

$$\begin{aligned} L_\sigma = \sum_j M_j^2 \mathcal{E}^{-1}[H_j^m, \Sigma_j] + m_j^2 M_j^2 \Big\{ & (H_{j+1}^m - H_j^m + \bar{D}_{\dot\alpha} L_\alpha^j + \cdots_{NL})^2 + \\ & + \frac{9}{4}\left(P_{j+1} - P_j - \frac{1}{3} D_\alpha L_\alpha^j + \cdots_{NL}\right)^2 + \cdots_j \Big\}, \end{aligned}$$

where the $\cdots_{NL}$ are the higher order, nonlinear, parts of SUGC transformations (cf. eqns. (5.22) and (5.24)). Now the lagrangian has the full SUGC^N symmetry under which the L_α^j transform as bifundamentals. Explicitly (or rather, implicitly), the transformations are:

$$e^{2iH_j} \to e^{\bar{\varepsilon}_j} e^{2iH_j} e^{-\varepsilon_j} \tag{7.3}$$
$$e^{\Lambda_j[L_j^\alpha]} \to e^{\varepsilon_j} e^{\Lambda_j[L_j^\alpha]} e^{-\varepsilon_{j+1}} . \tag{7.4}$$

We are using the notation of section 5: the operators ε_j, H_j and Λ_j are all supervector fields, (e.g. $H = H^m\partial_m$) and $\Lambda_j[L_j^\alpha]$ invokes the spinor superfield parameterization; ε_j is the SUGC on site j. We emphasize that there are only one set of coordinates $x, \theta, \bar{\theta}$ and all the transformations are active transformations under which the fields transform and the coordinates stay put.

The lagrangians L_U and L_σ present the same physical theory, in different gauges. Indeed, L_σ reduces to L_U after we use the extra symmetry to go to the gauge $L_\alpha^j = 0$. But the sigma model formulation is useful because we can now see why the UV completion should respect locality in theory space: it must preserve the SUGC^N symmetry. This symmetry forbids tree-level contact terms involving Φ_1 and Φ_N. Of course, these terms are generated through quantum corrections, but those are easy to estimate. There are a few types of corrections, which are analogous to the corrections in the well-studied gauge theory case [9].

First, there is the contribution from UV divergent gravity loops. Note that we can decouple any of the links by taking $M_j \to \infty$ holding $\Lambda_5^j = M_j m_j^4$ fixed. This works because the only interactions with Λ_5 strength are among the chiral goldstones G^j, coming from $L_\alpha^j = D_\alpha G^j + \cdots$ which do not mix sites. The proper procedure, according to naive dimensional analysis is to include all the operators coupling the links with their proper strengths. As we saw in the previous section (see also [12]), these have the form

$$\Lambda^2 \left(\hat{e}_j \frac{\hat{H}^c_{j+1} - \hat{H}^c_j}{\Lambda} \right)^p \left(\frac{\partial}{\Lambda} \right)^q \left(\frac{\hat{G}^c_j}{\Lambda} \right)^p , \tag{7.5}$$

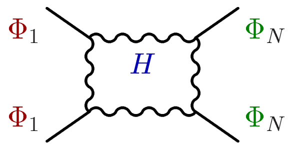
where $\hat{H}^c$ and $\hat{G}^c$ are the canonically normalized metric and goldstone fields and

$$\hat{e}_j \equiv \left(\frac{m_j}{M_j} \right)^{2/5} . \tag{7.6}$$

Once we use a self-consistent form of the links, we see that each vertex connecting nearest neighbors will have at least one factor of $\hat{e}$. A standard spurion argument then shows that the UV contribution is at most

$$\frac{\Lambda^2}{M_{\rm Pl}^4} \hat{e}^{2(N-1)} \Phi_1^2 \Phi_N^2 . \tag{7.7}$$

The second quantum contribution we are concerned with also comes from gravity loops, but it is saturated in the IR and completely finite. It is easy to see why the 1-loop contribution cannot vanish completely. If we run down to low energies, below the mass of the first KK mode, $m_1 \sim m/N \equiv \frac{1}{R}$ the theory just appears to be 4D supergravity coupled to the chiral fields Φ_1 and Φ_N, with a cutoff at $1/R$. Since a 4D gravity loop diagram, such as



(7.8)

knows nothing about locality in theory space, dimensional analysis with cutoff $1/R$ tells us that this operator must appear with coefficient

$$\frac{1}{16\pi^2 M_{\rm Pl}^4 R^2}\Phi_1^2\Phi_N^2\,. \tag{7.9}$$

The amazing thing about deconstruction is that locality in theory space guarantees that these diagrams really are cutoff at $1/R$ in the full theory (7.2).[3] That is, the massive supergravity modes regulate the divergences. Essentially, we know the diagrams are cut-off because the only field configurations which contribute are non-local. In 5D the operator requires a Wilson loop going around the whole space, and in deconstruction it requires extended field configurations which can be understood with the Coleman-Weinberg potential (again, see [9]).

Now, there is one more way the operator $\Phi_1^2\Phi_N^2$ might appear. Since the theory space lagrangian L_U (7.2) is non-renormalizable, we must imagine that it is eventually embedded in a UV completion. Already, the cutoff dependence of (7.7) indicates that the gravity sector is sensitive to UV effects and it is certainly consistent within the effective theory to take this as the only contribution. But, by analogy with extra dimensional theories, we can also imagine that there are "bulk" states near or slightly above the cutoff, $M_{\rm bulk} \gtrsim \Lambda$, which couple to both site 1 and site N (or both branes in a continuum 5D theory), and to Φ_1 and Φ_N. In theory space, bulk states correspond to new fields at each site, with standard nearest-neighbor hopping terms. Integrating these fields out at tree-level produces a third contribution to non-local operators:

$$\frac{1}{M_{\rm bulk}^2}\left(\frac{N}{RM_{\rm bulk}}\right)^{2(N-1)}\Phi_1^2\Phi_N^2\,. \tag{7.10}$$

In general, we expect $M_{\rm bulk}$ to be much larger than the inverse lattice spacing N/R, and so this contribution is negligible for large N. But for small N it may be important.

Finally, let us formally take the continuum limit of the lagrangian (7.2).[4] At the linearized level, we recover the 5D supergravity lagrangian of [19], without the radion field, and with our goldstones L_α representing their Ψ_α. Of course, this is exactly what we expect, as we can use 5D symmetries to set $\Psi_\alpha = 0$ and then restore these symmetries with goldstone fields after the dimensional reduction. Note that the $N = 2$ supersymmetry of 5D is non-linearly realized by the goldstones, as it must be because we have a consistent theory with two (Weyl) gravitini.

[3] With 2 sites there is a residual logarithmic divergence.

[4] This limit cannot actually be taken in a consistent quantum theory, see [13, 14]. Here we are just making observations about a correspondence between degrees of freedom at the classical level.

8. Two site anomaly mediation

As an example, to illustrate the simplicity of supergravity in theory space, we sketch a 2-site model of anomaly mediation. Anomaly mediation is a method of communicating supersymmetry breaking to the standard model. It relies on the fact that soft-masses for sfermions and gauginos are automatically generated in the presence of any supersymmetry-breaking sector that couples to gravity, by virtue of the scale anomaly of the standard model [6, 27]. Moreover, because the soft masses are related to the breaking of scale invariance, they are completely determined by the anomalous dimensions of the standard model fields.

The main advantage of anomaly-mediated supersymmetry breaking is that it solves the supersymmetric flavor problem. Because the scalar masses are insensitive to UV physics [6] and [27]–[29], the only flavor breaking spurions at low energy are the Yukawa matrices and flavor-changing neutral currents are naturally suppressed by the superGIM mechanism. For this solution to work, the anomaly mediated contribution must dominate other sources of scalar soft masses. In particular, soft masses generated in the UV where there is flavor physics at work are dangerous and need to be suppressed. Probably the simplest way of achieving this involves sequestering a hidden sector with an extra dimension; if the standard model is confined to one brane, and the supersymmetry breaking sector to another, dangerous operators are forbidden by locality. We can already see, by the considerations of the previous section, that all propitious features of sequestered sector anomaly mediation should be reproduced in theory space.

The simplest model has two sites:

$$\text{SM} \longrightarrow \text{hid}. \tag{8.1}$$

We put the standard model on site 1 and the hidden sector on site 2. Each site has its own supergravity multiplet, and the sites only interact through a supergravity link. In unitary gauge, where the link is eaten, the relevant part of the lagrangian is

$$\begin{aligned}\mathcal{L} = {} & \frac{1}{2}M_{\rm Pl}^2\mathcal{E}^{-1}[H_1^m,\Sigma_1] + \frac{1}{2}M_{\rm Pl}^2\mathcal{E}^{-1}[H_2^m,\Sigma_2] + \\ & + M_{\rm Pl}^2\frac{1}{R^2}\left\{(H_2^m - H_1^m)^2 + \frac{9}{4}(P_2 - P_1)^2 + \cdots_1\right\} + \\ & + (Q^\dagger Q + \ldots_1) + (S^\dagger S + \ldots_2)\,. \end{aligned} \tag{8.2}$$

Here we have only shown the Kahler potential part. Q stands for MSSM matter fields and S is a hidden sector field. The $\cdots_j$ are the additional terms in the Kahler potential required to make it invariant under the SUGC group on site j. This theory has a massless supergravity multiplet $H_1^m + H_2^m$ with Planck strength $M_{\rm Pl}$ and a massive supergravity multiplet $H_1^m - H_2^m$ of mass $1/R$. We use R only to make contact with the parameters of 5D anomaly mediation, as this 2-site model clearly has no continuum interpretation.

We assume that the hidden sector chiral superfield S gets an F-term vacuum expectation value signaling the breaking of supersymmetry. This is communicated to the visible sector just like in 5D anomaly mediation, through the massless supergravity mode, or equivalently the F term of its chiral compensator Σ: $\langle F_\Sigma \rangle \sim \langle F_S \rangle / M_{\rm Pl}$. Just as in 5D, the gravitational anomaly is completely independent of UV physics and so masses of the MSSM particles are proportional to their β functions. For example, sfermion masses are roughly:

$$m_s^2 \sim \left(\frac{\alpha}{4\pi}\right)^2 \langle F_\Sigma \rangle^2 \, . \tag{8.3}$$

And, like in 5D anomaly mediation, some of the sleptons will be tachyonic. We will not attempt to solve this problem here — we are merely replicating anomaly mediation in a two site model.

The next step is to consider quantum corrections which generate contact terms between the standard model and supersymmetry breaking sector. The biggest danger comes from soft masses coming out of contact terms like $Q^\dagger Q S^\dagger S$ generated by UV physics. But, as we have argued in the previous section, these operators are highly constrained by locality in theory space. In the 2-site model, the contribution from massive fields at the cutoff (cf. (7.10) with $M_{\rm bulk} \sim \Lambda$) is of order

$$\Delta L_{\rm bulk} \sim \frac{1}{R^2 \Lambda^4} Q^\dagger Q S^\dagger S \, . \tag{8.4}$$

And the contribution from gravity loops (cf. (7.7)) is

$$\Delta L_{\rm UV} \sim \frac{1}{M_{\rm Pl}^2} \left(\frac{1}{M_{\rm Pl} R}\right)^{12/5} Q^\dagger Q S^\dagger S \, . \tag{8.5}$$

If we take the cutoff to be around the strong coupling scale $\Lambda \sim (M_{\rm Pl} R^{-4})^{1/5}$ the gravity contribution is suppressed by powers of R. The bulk contribution also becomes suppressed with more sites, but the 2-site theory space need not contain heavy states at all, as it has no extra-dimensional interpretation.

In addition to UV contributions, there is also the finite IR contribution at one loop (cf. (7.9)):

$$\Delta L_{\rm IR} \sim \frac{1}{16\pi^2 M_{\rm Pl}^4 R^2} Q^\dagger Q S^\dagger S \, . \tag{8.6}$$

These operators can be important because they contribute to scalar masses:

$$\delta m_s^2 \propto \frac{1}{16\pi^2 M_{\rm Pl}^4 R^2} \langle F_S \rangle^2 \, . \tag{8.7}$$

The analogous continuum contribution was calculated recently in [30] and [31] and was found to be negative. We should expect the same sign for (8.7) and therefore it cannot be immediately used to ameliorate the problem of tachyonic sleptons, as one might have hoped.

9. Conclusions

In the first several sections of this paper, we presented and analyzed a natural interacting theory of massive supergravity. It is given by the lagrangian (1.1). On shell, this theory contains a massive supergravity multiplet containing a spin-2, a spin-1 and two spin 3/2 fields all degenerate in mass. The lagrangian is constructed using the 4D $N = 1$ superfield formalism, so it manifestly preserves global supersymmetry. We have demonstrated the validity of the lagrangian in three ways: first, with a formal analysis using spin projectors, second, using the explicit component expansion of both the bosonic and fermionic sectors, and finally using goldstones. This last method is particularly useful as it leads to an efficient way to study the interactions in the theory. Indeed, part of the justification of our lagrangian is that it contains only the fields of minimal supergravity, and so the interactions and (broken) symmetries of the theory can be simply lifted from the massless case. With other linearized lagrangians, such as the ones in [1], there are additional auxiliary fields whose symmetry properties are unknown. Therefore, working out the effect of the mass term on their interactions appears prohibitively difficult. Curiously, while attempting such a program we came across a mysterious alternative to the Fierz-Pauli lagrangian for a massive graviton invoking an auxiliary vector field (2.3). As far as we know, this lagrangian was not presented previously, and it may be of interest in its own right.

Returning to massive supergravity, we found by merging the goldstone formalism for gravity developed in [12] and the representation of supergeneral coordinate transformations from [23] that massive supergravity has the same scale of strong interactions as massive (non-supersymmetric) gravity. Namely, it breaks down at $\Lambda_5 = (M_{\rm Pl} m^4)^{1/5}$. We worked out the form of the strongest interactions in terms of a chiral multiplet of goldstones. This multiplet, in unitary gauge, comprises the scalar longitudinal mode of the graviton, the gravitino's chiral conjugate, and a scalar auxiliary field. One might describe it as the chiral supergoldstone of a real superfield containing the graviton's vector longitudinal polarizations, a Dirac goldstino, and a vector auxiliary field. In section 6 we simply called it G. The most critical feature of G is that it only gets a kinetic term from mixing with the metric superfield. The consequences of this mixing in supergravity are just as dire as for non-supersymmetric gravity: it leads to vDVZ type discontinuity as the mass is taken to zero; it causes the gravitational field around massive sources to break down at distances much larger than the Schwarzschild radius; and it prevents the reproduction of long distance supergravitational phenomena on a lattice. We have not actually demonstrated any of this, but the results follow trivially from the kinetic mixing and the logic in [12]. One might have hoped that the gravitino would miraculously cancel the troublesome amplitudes in graviton scattering which cause these effects, but this does not happen. One might also have hoped to unravel the strange holographic-type bounds which turned up in the lattice investigation of [13, 14], especially considering supersymmetry's role in consistent theories of quantum gravity. But again, supersymmetry is of no help.

On the brighter side, the lagrangian for massive supergravity naturally leads to theory space versions of sequestered sector models. In such models, visible and hidden sector fields are physically separated in an extra dimension, so that couplings between them are suppressed. We have shown that the same dangerous operators can be suppressed in a purely 4D theory by the addition of massive supergravity modes with prescribed couplings. Locality in theory space serves the same proscriptive function as locality in an extra dimension. We considered in particular a 2-site model of anomaly mediation.

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Working through the relevant corrections, by dimensional and symmetry analysis, we found that deconstruction can reproduce the phenomenological success (and failures) of a 5D anomaly mediated model.

Our goal in the analysis of deconstruction was to exhibit the correspondence between supergravity in 5D and in theory space, particularly in regard to issues of locality. In this vein, deconstruction provides a useful tool for studying extra dimensions. But it is also true that these theories have space to improve on the extra dimensional models, as 4D effective theories are less restrictive. For example, the radion can be simply thrown out. If the radion, or some other light field, ever proves to be useful in a compactified extra dimensional model, it can easily be co-opted into a theory space lagrangian. But it is equally possible that fields with no extra dimensional interpretation at all will prove advantageous.

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